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MSSM lightest CP-even Higgs boson mass to $\mathcal{O}(\alpha_s\alpha_t)$: the effective potential approach

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Abstract: Starting with the two-loop effective potential of the MSSM, and assuming a supersymmetric scale well above M_Z, we derive a simple analytical approximation for the lightest CP-even Higgs boson mass including resummation of higher order logarithmic terms via RG-improvement and finite non-logarithmic terms up to $\mathcal{O}(\alpha_s\alpha_t)$. This formula describes the most relevant radiative corrections to the MSSM Higgs boson mass, in particular, those associated with non-zero top-squark mixing.

Keywords: Supersymmetric Standard Model, Higgs Physics.

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Contents

1. Introduction

Supersymmetry (SUSY) stabilizes the hierarchy between the scale of electroweak symmetry breaking and the fundamental energy scale (the GUT, Planck or string scale) and makes plausible that this breaking occurs in a weakly coupled Higgs sector. The simplest realistic model that accommodates these ideas is the Minimal Supersymmetric Standard Model (MSSM), the paradigmatic testing ground of low-energy SUSY. Given a weakly coupled Higgs sector and a scale of electroweak symmetry breaking $v = 246\,\text{GeV}$, it follows [1] that the spectrum of the theory must contain an scalar particle h^0 with mass controlled by the Fermi scale and non-zero couplings to the $W^\pm$ and Z^0 gauge bosons (which is crucial for its detection at accelerators [2]). In the MSSM, the mass of this light Higgs boson (besides this Higgs particle, the Higgs sector in the MSSM contains a heavier scalar H^0, one pseudoscalar A^0 and a pair of charged Higgses $H^\pm$) is calculable. A precise determination of this mass is of paramount importance for the experimental search of SUSY. If the experimental lower bound on the Higgs boson mass [3] increases above the theoretical prediction we can rule out the MSSM. It is therefore understandable that this calculation has received a great deal of attention and attracted the efforts of many groups [4]–[10].

At tree level, the mass squared, $m^2_{h^0}$, of the light Higgs boson has an upper bound (saturated for large values of the pseudoscalar mass m_{A^0}) given by $M_Z^2 \cos^2 2\beta$. This is already below the experimental lower bound from LEP2 [3]. However, radiative corrections can raise the upper bound on $m^2_{h^0}$ dramatically [4]. The dominant contribution is

$$\Delta m^2_{h^0} = \frac{3m_t^4}{2\pi^2 v^2} \ln \frac{m^2_{\tilde{t}}}{m_t^2}\,, \tag{1.1}$$

where m_t is the top quark mass and $m_{\tilde{t}}$ a common top-squark mass. Radiative corrections to $m^2_{h^0}$ have been computed using different techniques: effective potential approach, direct diagrammatic calculation and effective theories with renormalization group (RG) tools. Each approach has its own virtues. The effective potential way simplifies the computation; the diagrammatic calculation is unavoidable to pick up some particular corrections; and the RG approach permits resummation of logarithmic terms to all loops and provides a physically meaningful organizing principle for the radiative corrections. There is no real need to choose one among these methods: the best way to proceed is to combine the three methods, taking advantage of the best virtue of each in turn.

The n-loop contribution to the (dimensionless) quantity $m^2_{h^0}/m_t^2$ has the schematic form

$$\sum_{k=0}^{n}\left(\alpha \ln\frac{m_{\tilde{t}}}{m_t}\right)^k \alpha^{n-k}\,, \tag{1.2}$$

where α represents the expansion parameter (with $\alpha_t = h_t^2/4\pi$ and $\alpha_s = g_s^2/4\pi$ giving the dominant contributions), and the logarithms $\ln(m_{\tilde{t}}/m_t)$ can be sizable for large values of $m_{\tilde{t}}$. The term $k=n$ in eq. (1.2) is the leading-logarithmic correction and dominates the n-loop contribution for large $m_{\tilde{t}}$. The term $k=n-1$ is the sub-leading logarithmic term, etc. Finally, the term $k=0$ gives the finite non-logarithmic piece of the n-loop correction. The complete one-loop radiative corrections to $m^2_{h^0}/m_t^2$ have been calculated. The dominant leading-logarithmic contribution is the $\alpha_t \ln(m_{\tilde{t}}/m_t)$ term in eq. (1.1). At this loop order there are no α_s corrections, which enter at two-loops. The most important finite corrections depend on the top-squark mixing parameter $X_t = A_t + \mu\cot\beta$, is given by

$$\Delta m^2_{h^0} = \frac{3m_t^4}{2\pi^2 v^2}\left(\frac{X_t^2}{m^2_{\tilde{t}}} - \frac{X_t^4}{12 m^4_{\tilde{t}}}\right), \tag{1.3}$$

and its effect on $m^2_{h^0}$ can be important. The maximum value for the upper bound on the Higgs mass is obtained for $X_t^2 = 6m^2_{\tilde{t}}$ (the so-called 'maximal-mixing' case).

The most important part of the higher order radiative corrections can be collected and resummed using renormalization group techniques [5]. Resumming with one-loop RG equations takes into account the leading-logarithmic corrections to all loops, and using two-loop RG equations sub-leading logarithms are also included. However, the finite non-logarithmic terms cannot be obtained in this way. Getting them at two-loops requires a genuine two-loop calculation. This was first done, in some particularly simple limit, in ref. [6], but the effect of non-zero top-squark mixing was not included. Recently, a two-loop diagrammatic computation to $\mathcal{O}(\alpha_t\alpha_s)$ has been completed [8], and the effective potential has been also calculated to the same order [9]. These studies show that the most significant two-loop effect, not previously taken into account by RG techniques, comes from the finite pieces dependent on the top-squark mixing. The new refined upper bound on m_{h^0} in the region of maximal mixing can increase up to 5 GeV (if $m_{\tilde{t}} \sim 1\,\mathrm{TeV}$) and the condition for maximal-mixing itself gets also slightly modified.[1]

Making good use of RG resummation, it is possible to obtain compact analytical approximations for $m^2_{h^0}$ which take into account the most important radiative corrections [5]. Besides being of direct practical interest, these formulae are theoretically interesting as they provide a clear picture of the physical origin of the dominant contributions. In this paper we extract such an analytical approximation for $m^2_{h^0}$ starting with the two-loop effective potential computed in [9]. Our final formula includes the most important $\mathcal{O}(\alpha_s\alpha_t)$ radiative corrections, in particular the finite terms associated with non-zero top-squark mixing. Our results agree with those obtained by diagrammatic techniques [10] where they overlap, but are computed by an alternative way. We include, in addition, RG resummation which allows us to write a particularly simple final formula for the radiative corrections to $m^2_{h^0}$; this is the main result of our paper. Again, to the order at which previous RG results [5] were computed, we find agreement with our results. Our final formula for $m^2_{h^0}$ improves over previous RG formulae by including the genuine two-loop threshold corrections (which are important for large values of the top-squark mixing) and over previous diagrammatic results by incorporating RG-resummation of logarithmic corrections.

2. Two-loop effective potential in the MSSM

The two-loop effective potential in the MSSM to $\mathcal{O}(\alpha_s\alpha_t)$ has been calculated for the case of zero top-squark mixing and $\tan\beta = \infty$ in [6], where $\tan\beta = v_2/v_1$ is the usual ratio of the vacuum expectation values of the two Higgs fields; and for arbitrary top-squark mixings and $\tan\beta$ values in [9],[2]

$$\begin{aligned}(16\pi^2)^2\; V_2(h_1,h_2) = 8g_3^2\Big\{ & J(m_t,m_t) - 2m_t^2 I(m_t,m_t,0) + \frac{1}{2}(c_t^4+s_t^4)\sum_{i=1}^{2} J(m_{\tilde{t}_i},m_{\tilde{t}_i}) + \\ & + 2s_t^2c_t^2 J(m_{\tilde{t}_1},m_{\tilde{t}_2}) + \sum_{i=1}^{2} m_{\tilde{t}_i}^2 I(m_{\tilde{t}_i},m_{\tilde{t}_i},0) + 2J(m_{\tilde{g}},m_t) - \\ & - \sum_{i=1}^{2}\Big[J(m_{\tilde{t}_i},m_{\tilde{g}}) + J(m_{\tilde{t}_i},m_t) + \\ & \qquad + (m_{\tilde{t}_i}^2 - m_{\tilde{g}}^2 - m_t^2) I(m_{\tilde{t}_i},m_{\tilde{g}},m_t)\Big] - \\ & - 4m_{\tilde{g}}\, m_t\, s_t c_t \left[I(m_{\tilde{t}_1},m_{\tilde{g}},m_t) - I(m_{\tilde{t}_2},m_{\tilde{g}},m_t)\right]\Big\}, \end{aligned} \tag{2.1}$$

[1] This is not a disagreement between different calculations, but the result of including in the analysis of [8] two-loop effects inaccessible to previous approaches.

[2] We choose to work in the $\overline{\mathrm{DR}}$ scheme [11], as required by preserving the SUSY Ward identities. This choice also simplifies complications associated with vector-boson loops in the $\overline{\mathrm{MS}}$ scheme. The two-loop effective potential in eq. (2.1) is calculated in the Laudau gauge, but it is also correct for a general R_ξ gauge.

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where the arguments h_1, h_2 are the neutral scalar components of the Higgs fields H_1 and H_2. All the masses and mixing angles in eq. (2.1) should be understood as h_1, h_2-dependent quantities, for example, the top quark mass $m_t = \frac{1}{\sqrt{2}} h_t h_2$. We use short-hand notations $c_t = \cos\theta_{\tilde{t}}$, $s_t = \sin\theta_{\tilde{t}}$, where $\theta_{\tilde{t}}$ is the top-squark mixing angle. The minimally subtracted[3] two-loop scalar functions I and J are [12]:

$$
\begin{aligned}
I(m_1, m_2, m_3) = -\frac{1}{2}\Bigg[& (-m_1^2 + m_2^2 + m_3^2) \ln\frac{m_2^2}{Q^2} \ln\frac{m_3^2}{Q^2} + \\
& + (m_1^2 - m_2^2 + m_3^2) \ln\frac{m_1^2}{Q^2} \ln\frac{m_3^2}{Q^2} + \\
& + (m_1^2 + m_2^2 - m_3^2) \ln\frac{m_1^2}{Q^2} \ln\frac{m_2^2}{Q^2} - \\
& - 4\left(m_1^2 \ln\frac{m_1^2}{Q^2} + m_2^2 \ln\frac{m_2^2}{Q^2} + m_3^2 \ln\frac{m_3^2}{Q^2} \right) + \\
& + \xi(m_1, m_2, m_3) + 5(m_1^2 + m_2^2 + m_3^2) \Bigg] \\
J(m_1, m_2) = m_1^2 m_2^2 \Bigg[& 1 - \ln\frac{m_1^2}{Q^2} - \ln\frac{m_2^2}{Q^2} + \ln\frac{m_1^2}{Q^2} \ln\frac{m_2^2}{Q^2} \Bigg], \qquad (2.2)
\end{aligned}
$$

where Q is the renormalization scale. The function $\xi(m_1, m_2, m_3)$ in the expression of $I(m_1, m_2, m_3)$ has been calculated in [12] by a differential equation method and in [13] by a Mellin-Barnes integral representation method. Its final form can be expressed in terms of Lobachevsky's functions or Clausen's integral functions and their analytical continuations.

It is important to check that the effective potential $V(h_1, h_2) = V_0(h_1, h_2) + V_1(h_1, h_2) + V_2(h_1, h_2)$ is invariant under changes of the renormalization scale up to higher order terms of $\mathcal{O}(\alpha_t^2)$. To prove this, we write the tree-level and one-loop potential as follows (neglecting contributions from the gauge boson, Higgs/Goldstone boson and neutralino/chargino sectors):

$$
\begin{aligned}
V_{0+1}(h_1, h_2) = & \frac{1}{2}(m_{H_1}^2 + \mu^2) h_1^2 + \frac{1}{2}(m_{H_2}^2 + \mu^2) h_2^2 + B_\mu h_1 h_2 + \\
& + \frac{3}{16\pi^2}\Big[G(m_{\tilde{t}_1}) + G(m_{\tilde{t}_2}) - 2G(m_t) \Big], \qquad (2.3)
\end{aligned}
$$

where m_{H_1}, m_{H_2} and B_μ are the soft-breaking Higgs sector mass parameters, μ the supersymmetric Higgs-boson mass parameter, and

$$
G(m) = \frac{m^4}{2}\left(\ln\frac{m^2}{Q^2} - \frac{3}{2} \right). \qquad (2.4)
$$

[3]Subtraction of one-loop sub-divergences can be treated following the first reference of [12]. In obtaining eq. (2.1), we have also added freely one-loop diagrams depending only on $m_{\tilde{g}}$, as they do not affect the Higgs boson mass calculation in the effective potential approach.

It is then straightforward to show that

$$\mathcal{D}^{(2)}V_0 = -\frac{\partial V_2}{\partial \ln Q^2} - \mathcal{D}^{(1)}V_1 = \frac{8g_3^2h_t^2h_2^2}{(16\pi^2)^2}\left(M_{\widetilde{Q}}^2 + M_{\widetilde{U}}^2 + 2M_3^2 + X_t^2 - 2M_3X_t\right), \quad (2.5)$$

modulo terms independent of the Higgs field h_2, and where $X_t = A_t + \mu\cot\beta$, with A_t a trilinear coupling (with dimensions of mass) appearing in the following term of the soft-breaking Lagrangian: $h_tA_tH_2\widetilde{Q}\widetilde{U}$. $M_{\widetilde{Q}}$ and $M_{\widetilde{U}}$ are soft-breaking masses for the left- and right-handed top squarks, $\widetilde{Q}$ and $\widetilde{U}$, respectively, and M_3 is the gluino soft mass. (It is a nontrivial check that all $\ln Q^2$ terms cancel with each other in eq. (2.5)). In the above equation $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(2)}$ stand for one- and two-loop RG variations of the parameters, and we have used the following equations

$$\begin{aligned}
\frac{\partial I(m_1,m_2,m_3)}{\partial \ln Q^2} &= m_1^2\ln\frac{m_1^2}{Q^2} + m_2^2\ln\frac{m_2^2}{Q^2} + m_3^2\ln\frac{m_3^2}{Q^2} - 2(m_1^2+m_2^2+m_3^2)\,, \\
\frac{\partial J(m_1,m_2)}{\partial \ln Q^2} &= m_1^2m_2^2\left(2 - \ln\frac{m_1^2}{Q^2} - \ln\frac{m_2^2}{Q^2}\right), \qquad (2.6)
\end{aligned}$$

and the MSSM RG equations from ref. [15],

$$\begin{aligned}
\frac{\partial m_{H_2}^2}{\partial \ln Q^2} &= \frac{3h_t^2}{16\pi^2}(m_{H_2}^2 + M_{\widetilde{Q}}^2 + M_{\widetilde{U}}^2 + A_t^2) + \\
&\quad + \frac{16g_3^2h_t^2}{(16\pi^2)^2}(m_{H_2}^2 + M_{\widetilde{Q}}^2 + M_{\widetilde{U}}^2 + A_t^2 + 2M_3^2 - 2M_3A_t)\,, \\
\frac{\partial \ln\mu}{\partial \ln Q^2} &= -\frac{\partial \ln h_2}{\partial \ln Q^2} = \frac{3h_t^2}{32\pi^2} + \frac{8g_3^2h_t^2}{(16\pi^2)^2}\,, \\
\frac{\partial B_\mu}{\partial \ln Q^2} &= \frac{3h_t^2}{16\pi^2}\left(\frac{B_\mu}{2} + A_t\mu\right) + \frac{16g_3^2h_t^2}{(16\pi^2)^2}\left(\frac{B_\mu}{2} + A_t\mu - M_3\mu\right). \qquad (2.7)
\end{aligned}$$

We will see that the RG does not only provide an important check for the Higgs boson mass correction formulae in the later sections, but also allows us to present those formulae in a more physically appealing form−the RG improved form.

Using the effective potential in eq. (2.1), two-loop radiative corrections to $m_{h^0}^2$ (to the order of $\mathcal{O}(\alpha_s\alpha_t)$) have been calculated numerically in [9]. In the following, we shall derive an approximation formula valid for $M_{\widetilde{Q}}(= M_{\widetilde{U}})$ and $m_{A^0} \gg M_Z$, where m_{A^0} is the mass of the pseudoscalar Higgs boson A^0.

3. Analytical expression for $\Delta m_{h^0}^2$ from the effective potential

To simplify our analytical expression, we assume the squark soft masses satisfy $M_{\widetilde{Q}} = M_{\widetilde{U}} \equiv M_S$, where M_S is the SUSY scale. The two eigenvalues and mixing angle of top-squark squared-mass matrix are

$$m_{\tilde{t}_1}^2 = m_{\tilde{t}}^2 + m_tX_t\,, \qquad m_{\tilde{t}_2}^2 = m_{\tilde{t}}^2 - m_tX_t\,, \qquad s_t = c_t = \frac{1}{\sqrt{2}}\,, \qquad (3.1)$$

where the average top-squark mass $m_{\tilde{t}}^2 = M_S^2 + m_t^2$. We shall further assume the gluino mass $m_{\tilde{g}} = M_S$.

Under these simplifications, the effective potential to $\mathcal{O}(\alpha_s \alpha_t)$ two-loop order is (neglecting one-loop sub-dominant terms)

$$
\begin{aligned}
V(h_1,h_2) = V_0(h_1,h_2) &+ \frac{3}{16\pi^2}\left[2G(m_{\tilde{t}}) - 2G(m_t) + m_t^2 X_t^2 \ln\frac{m_{\tilde{t}}^2}{Q^2} - \frac{m_t^4 X_t^4}{12 m_{\tilde{t}}^4}\right] + \\
&+ \frac{8g_3^2}{(16\pi^2)^2}\Bigg\{ J(m_t,m_t) - 2m_t^2 I(m_t,m_t,0) + J(m_{\tilde{t}},m_{\tilde{t}}) + 2m_{\tilde{t}}^2 I(m_{\tilde{t}},m_{\tilde{t}},0) + \\
&\qquad + 2J(M_S,m_t) - 2J(M_S,m_{\tilde{t}}) - 2J(m_t,m_{\tilde{t}}) + \\
&\qquad + m_t^2\left[2X_t m_{\tilde{t}}\left(1-\ln\frac{m_{\tilde{t}}^2}{Q^2}\right)^2 + X_t^2\left(1-\ln^2\frac{m_{\tilde{t}}^2}{Q^2}\right)\right] + \\
&\qquad + m_t^4\left[-\frac{X_t}{m_{\tilde{t}}}\left(\left(1-\ln\frac{m_{\tilde{t}}^2}{Q^2}\right)^2 + 2\ln\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{X_t^2}{m_{\tilde{t}}^2} + \right. \\
&\qquad\qquad \left. + \frac{X_t^3}{3m_{\tilde{t}}^3}\left(1-2\ln\frac{m_{\tilde{t}}^2}{Q^2}\right) - \frac{X_t^4}{12 m_{\tilde{t}}^4}\right]\Bigg\}, \qquad (3.2)
\end{aligned}
$$

where we have separated terms depending on X_t (to make the top-squark mixing effects transparent) and expanded the effective potential in powers of $m_t/m_{\tilde{t}}$ (and $m_t X_t/m_{\tilde{t}}^2$) with higher order terms in $m_t/m_{\tilde{t}}$ neglected. It is now straightforward to find the corrections to the four entries in the Higgs boson squared-mass matrix[4]

$$
\begin{aligned}
\Delta\mathcal{M}_{11}^2 &= h_1 \frac{\partial}{\partial h_1}\left(\frac{1}{h_1}\frac{\partial V(h_1,h_2)}{\partial h_1}\right)\Bigg|_{h_1=v_1,\,h_2=v_2} \\
&= \frac{\alpha_t}{4\pi}\left\{3\tan\beta\left[-\mu A_t \ln\frac{m_{\tilde{t}}^2}{Q^2} + \frac{m_t^2 \mu A_t X_t^2}{6 m_{\tilde{t}}^4}\right] - \frac{m_t^2\mu^2 X_t^2}{m_{\tilde{t}}^4}\right\} + \\
&+ \frac{\alpha_s\alpha_t}{2\pi^2}\Bigg\{-\mu\tan\beta\left[m_{\tilde{t}}\left(1-\ln\frac{m_{\tilde{t}}^2}{Q^2}\right)^2 + A_t\left(1-\ln^2\frac{m_{\tilde{t}}^2}{Q^2}\right)\right] + \\
&\qquad + \frac{m_t^2\mu}{2m_{\tilde{t}}}\tan\beta\left[\left(1-\ln\frac{m_{\tilde{t}}^2}{Q^2}\right)^2 + 2\ln\frac{m_{\tilde{t}}^2}{m_t^2}\right] - \frac{m_t^2\mu A_t}{m_{\tilde{t}}^2}\tan\beta + \\
&\qquad + \frac{m_t^2 X_t}{m_{\tilde{t}}^3}\left(\mu^2 - \frac{\mu X_t}{2}\tan\beta\right)\left[1-2\ln\frac{m_{\tilde{t}}^2}{Q^2}\right] + \\
&\qquad + \frac{m_t^2\mu X_t^2}{2m_{\tilde{t}}^4}\left[-\mu + \frac{X_t}{3}\tan\beta\right]\Bigg\} \qquad (3.3)
\end{aligned}
$$

[4]The effective potential approach evaluates these corrections at the zero external momentum limit. This is consistent with our other approximations since the tree-level light Higgs boson mass is related to the electroweak gauge couplings and can be neglected.

$$
\begin{aligned}
\Delta\mathcal{M}^2_{12} = \Delta\mathcal{M}^2_{21} = &\left.\frac{\partial^2 V(h_1,h_2)}{\partial h_1 \partial h_2}\right|_{h_1=v_1, h_2=v_2} \\
= &\frac{3\alpha_t}{4\pi}\left[\mu A_t \ln\frac{m_{\tilde t}^2}{Q^2} + \frac{2m_t^2\mu X_t}{m_{\tilde t}^2} - \frac{m_t^2\mu A_t X_t^2}{2m_{\tilde t}^4}\right] + \\
&+\frac{\alpha_s\alpha_t}{2\pi^2}\left\{\mu\left[m_{\tilde t}\left(1-\ln\frac{m_{\tilde t}^2}{Q^2}\right)^2 + A_t\left(1-\ln^2\frac{m_{\tilde t}^2}{Q^2}\right)\right] + \right. \\
&\qquad + \frac{m_t^2\mu}{m_{\tilde t}}\left[-\frac{5}{2} + 5\ln\frac{m_{\tilde t}^2}{Q^2} - \frac{1}{2}\ln^2\frac{m_{\tilde t}^2}{Q^2} - 3\ln\frac{m_{\tilde t}^2}{m_t^2}\right] + \\
&\qquad + \frac{m_t^2\mu}{m_{\tilde t}^2}\left[A_t + 2X_t - 4X_t\ln\frac{m_{\tilde t}^2}{Q^2}\right] + \\
&\qquad \left. + \frac{m_t^2 X_t}{m_{\tilde t}^3}(\mu A_t + \frac{\mu X_t}{2})\left[1 - 2\ln\frac{m_{\tilde t}^2}{Q^2}\right] - \frac{m_t^2\mu A_t X_t^2}{2m_{\tilde t}^4}\right\}
\end{aligned} \tag{3.4}
$$

$$
\begin{aligned}
\Delta\mathcal{M}^2_{22} = &\left. h_2\frac{\partial}{\partial h_2}\left(\frac{1}{h_2}\frac{\partial V(h_1,h_2)}{\partial h_2}\right)\right|_{h_1=v_1,\, h_2=v_2} \\
= &\frac{3\alpha_t}{\pi}m_t^2\ln\frac{m_{\tilde t}^2}{m_t^2} + \\
&+\frac{\alpha_t}{4\pi}\left\{3\cot\beta\left[-\mu A_t\ln\frac{m_{\tilde t}^2}{Q^2} + \frac{m_t^2\mu A_t X_t^2}{6m_{\tilde t}^4}\right] + \left[\frac{12A_tX_t}{m_{\tilde t}^2} - \frac{A_t^2X_t^2}{m_{\tilde t}^4}\right]\right\} + \\
&+\frac{2\alpha_s\alpha_t}{\pi^2}m_t^2\left[\ln^2\frac{m_{\tilde t}^2}{m_t^2} - 2\ln^2\frac{m_{\tilde t}^2}{Q^2} + 2\ln^2\frac{m_t^2}{Q^2} + \ln\frac{m_{\tilde t}^2}{m_t^2} - 1\right] + \\
&+\frac{\alpha_s\alpha_t}{2\pi^2}\left\{-\mu\cot\beta\left[m_{\tilde t}\left(1-\ln\frac{m_{\tilde t}^2}{Q^2}\right)^2 + A_t\left(1-\ln^2\frac{m_{\tilde t}^2}{Q^2}\right)\right] + \right. \\
&\qquad + \frac{m_t^2}{m_{\tilde t}}\left[\frac{\mu}{2}\cot\beta\left(\left(1-\ln\frac{m_{\tilde t}^2}{Q^2}\right)^2 + 2\ln\frac{m_{\tilde t}^2}{m_t^2}\right) + \right. \\
&\qquad\qquad \left. + 2(A_t+X_t)\left(4\ln\frac{m_{\tilde t}^2}{Q^2} - 2\ln\frac{m_{\tilde t}^2}{m_t^2}\right) - 4A_t\right] + \\
&\qquad + \frac{m_t^2 A_t}{m_{\tilde t}^2}\left[4X_t - \mu\cot\beta - 8X_t\ln\frac{m_{\tilde t}^2}{Q^2}\right] + \\
&\qquad + \frac{m_t^2 X_t}{m_{\tilde t}^3}\left[A_t^2 - \frac{\mu X_t}{2}\cot\beta + \frac{X_t^2}{3}\right]\left[1 - 2\ln\frac{m_{\tilde t}^2}{Q^2}\right] + \\
&\qquad \left. + \frac{m_t^2 A_t X_t^2}{2m_{\tilde t}^4}\left[-A_t + \frac{X_t}{3}\right]\right\},
\end{aligned} \tag{3.5}
$$

where $\alpha_s = g_3^2/4\pi$ and $\alpha_t = h_t^2/4\pi$. Again we have neglected terms in higher orders of $m_t/m_{\tilde t}$.

The Higgs boson mass to the two-loop order can be obtained by diagonalizing the CP-even Higgs boson squared-mass matrix $\mathcal{M}^2$ by including these corrections

$$\mathcal{M}^2 = \begin{pmatrix} M_Z^2\cos^2\beta + m_{A^0}^2\sin^2\beta + \Delta\mathcal{M}_{11}^2 & -(M_Z^2+m_{A^0}^2)\sin\beta\cos\beta + \Delta\mathcal{M}_{12}^2 \\ -(M_Z^2+m_{A^0}^2)\sin\beta\cos\beta + \Delta\mathcal{M}_{21}^2 & M_Z^2\sin^2\beta + m_{A^0}^2\cos^2\beta + \Delta\mathcal{M}_{22}^2 \end{pmatrix}. \tag{3.6}$$

This means that the mixing angles at two-loop order are different from those at tree-level. Nevertheless, this difference is small to a good approximation and we can therefore use the tree-level mixing angle α to compute the Higgs boson mass. We further simplify the mass correction formula by approximating $\cos\alpha = \sin\beta$ and $\sin\alpha = -\cos\beta$, which are valid in the limit $m_{A^0} \gg M_Z$. The final analytical expression for the two-loop order correction to the lightest CP-even Higgs boson mass is

$$\begin{aligned}\Delta m_{h^0}^2 = {} & \frac{3m_t^4}{2\pi^2 v^2}\left[\ln\frac{m_{\tilde t}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde t}^2} - \frac{X_t^4}{12 m_{\tilde t}^4}\right] + \\ & + \frac{\alpha_s m_t^4}{\pi^3 v^2}\left\{\ln^2\frac{m_{\tilde t}^2}{m_t^2} - 2\ln^2\frac{m_{\tilde t}^2}{Q^2} + 2\ln^2\frac{m_t^2}{Q^2} + \ln\frac{m_{\tilde t}^2}{m_t^2} - 1 + \right. \\ & \qquad + \frac{X_t}{m_{\tilde t}}\left[-1 + 2\ln\frac{m_{\tilde t}^2}{Q^2} + 2\ln\frac{m_t^2}{Q^2}\right] + \\ & \qquad \left. + \left(\frac{X_t^2}{m_{\tilde t}^2} + \frac{X_t^3}{3m_{\tilde t}^3}\right)\left[1 - 2\ln\frac{m_{\tilde t}^2}{Q^2}\right] - \frac{X_t^4}{12 m_{\tilde t}^4}\right\}.\end{aligned} \tag{3.7}$$

We note that all parameters in eq. (3.7) are running parameters evaluated in the $\overline{\text{DR}}$ -scheme and satisfy MSSM RG equations. It is also easy to check that $\Delta m_{h^0}^2$ is indeed two-loop RG invariant up to terms of order $\mathcal{O}(\alpha_t^2)$ by using the following equations

$$\frac{\partial \ln m_t^2}{\partial \ln Q^2} = \frac{\partial \ln m_{\tilde t}^2}{\partial \ln Q^2} = -\frac{4\alpha_s}{3\pi}, \qquad \frac{\partial X_t}{\partial \ln Q^2} = \frac{4\alpha_s}{3\pi} M_3, \tag{3.8}$$

with the gluino soft mass $M_3 = m_{\tilde g} = M_S$ in our approximation.

At first sight, eq. (3.7) seems quite different from those formulae obtained by the RG improved one-loop effective potential approach [5] and by the two-loop diagrammatic approach [10]. The difference can be settled by observing that the parameters in eq. (3.7) are MSSM running parameters, while the top quark mass in [5] is a SM running mass and X_t and $m_{\tilde t}$ in [10] are on-shell (OS) parameters. Our one-loop parameters receive radiative corrections from the SM as well as SUSY particles, so they are different from the parameters in other approaches where the SUSY particles are explicitly decoupled. Furthermore, part of the difference arises from converting parameters in the $\overline{\text{DR}}$ -scheme to the $\overline{\text{MS}}$ -scheme.

To demonstrate this point, we first compare our result in eq. (3.7) with that of ref. [10]. We use the following relations of $\overline{\text{DR}}$ top quark mass (m_t) and top-squark running mass ($m_{\tilde t}$) (in the MSSM) and the $\overline{\text{MS}}$ running top quark mass ($\overline{m}_t$)

and OS top-squark mass ($\widehat{m}_{\tilde{t}}$) (in the SM), which for example can be deduced from refs. [7, 14]

$$m_t(Q) = \overline{m}_t(Q)\left[1 - \frac{\alpha_s}{3\pi}\left(1 + \ln\frac{m_{\tilde{t}}^2}{Q^2} - \frac{X_t}{m_{\tilde{t}}}\right)\right], \tag{3.9}$$

$$m_{\tilde{t}}(Q) = \widehat{m}_{\tilde{t}}\left[1 - \frac{4\alpha_s}{3\pi} + \frac{2\alpha_s}{3\pi}\ln\frac{m_{\tilde{t}}^2}{Q^2}\right], \tag{3.10}$$

up to higher order terms in $m_t/m_{\tilde{t}}$.[5] All terms in eqs. (3.9) and (3.10) come from the decoupling top squarks and gluinos, except for the finite term $-\alpha_s/3\pi$ in eq. (3.9), which is a $\overline{\mathrm{DR}}$ -$\overline{\mathrm{MS}}$ -scheme conversion factor. Similarly, one can relate X_t between different approaches by considering the top-squark/gluino contributions to the self energy $\Pi_{\tilde{t}_L\tilde{t}_R}$ [7], which gives

$$\begin{aligned} X_t(Q) &= \widehat{X}_t + \frac{1}{m_t}\left[\mathrm{Re}\Pi_{\tilde{t}_L\tilde{t}_R}(p^2 = m_{\tilde{t}}^2) - X_t\mathrm{Re}\Sigma_t(p^2 = m_t^2)\right] \\ &= \widehat{X}_t + \frac{\alpha_s}{3\pi}\left[m_{\tilde{t}}\left(8 - 4\ln\frac{m_{\tilde{t}}^2}{Q^2}\right) + X_t\left(5 + 3\ln\frac{m_t^2}{m_{\tilde{t}}^2}\right) - \frac{X_t^2}{m_{\tilde{t}}}\right], \end{aligned} \tag{3.11}$$

where $\widehat{X}_t$ is the OS top-squark mixing parameter. The external momentum of $\Pi_{\tilde{t}_L\tilde{t}_R}$ has been set to the average top-squark mass $m_{\tilde{t}}$.[6] The top-quark self energy Σ_t is evaluated in the MSSM and contains QCD corrections from top-squark/gluino and top/gluon loops (it can be deduced from refs. [7, 14]),

$$\mathrm{Re}\Sigma_t(p^2 = m_t^2) = -\frac{\alpha_s}{3\pi}m_t\left[5 - 3\ln\frac{m_t^2}{Q^2} + \ln\frac{m_{\tilde{t}}^2}{Q^2} - \frac{X_t}{m_{\tilde{t}}}\right]. \tag{3.12}$$

We note that in the effective SM the average top-squark mass $\widehat{m}_{\tilde{t}}$ and the parameter $\widehat{X}_t$ are frozen and equivalent to their physical 'pole' values since all SUSY particles have been decoupled.

Substituting eqs. (3.9), (3.10) and (3.11) into eq. (3.7), we obtain the mass correction formula

$$\begin{aligned} \Delta m_{h^0}^2 = & \frac{3\overline{m}_t^4}{2\pi^2 v^2}\left[\ln\frac{\widehat{m}_{\tilde{t}}^2}{\overline{m}_t^2} + \frac{\widehat{X}_t^2}{\widehat{m}_{\tilde{t}}^2} - \frac{1}{12}\frac{\widehat{X}_t^4}{\widehat{m}_{\tilde{t}}^4}\right] + \\ & + \frac{\alpha_s m_t^4}{\pi^3 v^2}\left\{-4 - 3\ln^2\frac{m_t^2}{Q^2} + 3\ln^2\frac{m_{\tilde{t}}^2}{Q^2} + 3\ln\frac{m_{\tilde{t}}^2}{Q^2} - \ln\frac{m_{\tilde{t}}^2}{m_t^2} + \right. \\ & \left. + \frac{6X_t}{m_{\tilde{t}}} + \frac{X_t^2}{m_{\tilde{t}}^2}\left[8 - 3\ln\frac{m_t^2}{Q^2} - 3\ln\frac{m_{\tilde{t}}^2}{Q^2}\right] + \frac{X_t^4}{m_{\tilde{t}}^4}\left[-\frac{17}{12} + \frac{1}{2}\ln\frac{m_{\tilde{t}}^2}{Q^2}\right]\right\}, \end{aligned} \tag{3.13}$$

where we have explicitly indicated the definition used for different parameters in the one-loop corrections; different definitions for the parameters in the two-loop

[5] Parameters inside the brackets can be running or OS ones, as the difference is of higher order.

[6] Choosing $p^2 = m_{\tilde{t}}^2 \pm m_t X_t$ changes the result by terms of higher order in $m_t/m_{\tilde{t}}$.

corrections would give differences of higher order. This equation is identical to the mass correction formula in [10] for $Q = m_t$. Comparison of our formula with the one obtained using the RG-improved one-loop potential [5] is left for the end of next section.

4. RG-improved Higgs boson mass

The final expression for $m^2_{h^0}$ in the last section was very convenient to make contact with the diagrammatic results of [10]. However, a simpler and more transparent expression can be obtained by transforming to the RG language. Furthermore, the improvement of the formula goes beyond purely aesthetic reasons, as it resums higher order corrections as well.

The idea is to let all parameters in the formula for $m^2_{h^0}$ be running parameters and to choose the scale at which they are evaluated in such a way that higher order logarithmic corrections are automatically taken care of. Moreover, the scale at which each parameter has to be evaluated to achieve this, is susceptible of physical interpretation.

Starting from eq. (3.7), we first use eq. (3.9) to translate the MSSM top-quark running mass $m_t(Q)$ into that of the SM, $\overline{m}_t(Q)$. We then use the solutions to the SM top quark mass RG equation and the MSSM RG equations of $m_{\tilde{t}}$ and X_t in eq. (3.8),

$$\overline{m}_t^2(Q) = \overline{m}_t^2(Q') \left[1 + \frac{2\alpha_s}{\pi} \ln \frac{Q'^2}{Q^2}\right], \tag{4.1}$$

$$m_{\tilde{t}}^2(Q) = m_{\tilde{t}}^2(Q') \left[1 + \frac{4\alpha_s}{3\pi} \ln \frac{Q'^2}{Q^2}\right], \tag{4.2}$$

$$X_t(Q) = X_t(Q') - \frac{4\alpha_s}{3\pi} M_3 \ln \frac{Q'^2}{Q^2}, \tag{4.3}$$

to relate their values at two different renormalization scales, Q and Q'. Our final formula is

$$\begin{aligned}\Delta m^2_{h^0} = {} & \frac{3}{2\pi^2 v^2}\left\{\overline{m}_t^4(Q_t) \ln \frac{m_{\tilde{t}}^2(Q_{\tilde{t}})}{\overline{m}_t^2(Q'_t)} + \overline{m}_t^4(Q_{\rm th}) \left[\frac{X_t^2(Q_{\rm th})}{m_{\tilde{t}}^2(Q_{\rm th})} - \frac{X_t^4(Q_{\rm th})}{12 m_{\tilde{t}}^4(Q_{\rm th})}\right]\right\} + \\ & + \frac{\alpha_s m_t^4}{\pi^3 v^2}\left[-\frac{2X_t}{m_{\tilde{t}}} - \frac{X_t^2}{m_{\tilde{t}}^2} + \frac{7}{3}\frac{X_t^3}{m_{\tilde{t}}^3} + \frac{1}{12}\frac{X_t^4}{m_{\tilde{t}}^4} - \frac{1}{6}\frac{X_t^5}{m_{\tilde{t}}^5}\right],\end{aligned} \tag{4.4}$$

with $Q_t^2 = m_t m_{\tilde{t}}$, $Q_{\tilde{t}}^2 m_t = Q'^3_t$ (satisfied by $Q_{\tilde{t}} = Q'_t = m_t$ or $Q_{\tilde{t}} = m_{\tilde{t}}$, $Q'^3_t = m_t m_{\tilde{t}}^2$) and $Q_{\rm th} = m_{\tilde{t}}$. Note how nicely everything falls into place. All higher order logarithms are reabsorbed into the one-loop RG-improved term and the only two-loop pieces remaining are the finite X_t-dependent terms.

The logarithmic term in eq. (4.4) can be interpreted as the result of integrating the RG equation for the quartic Higgs coupling between the high energy scale $m_{\tilde{t}}$ and

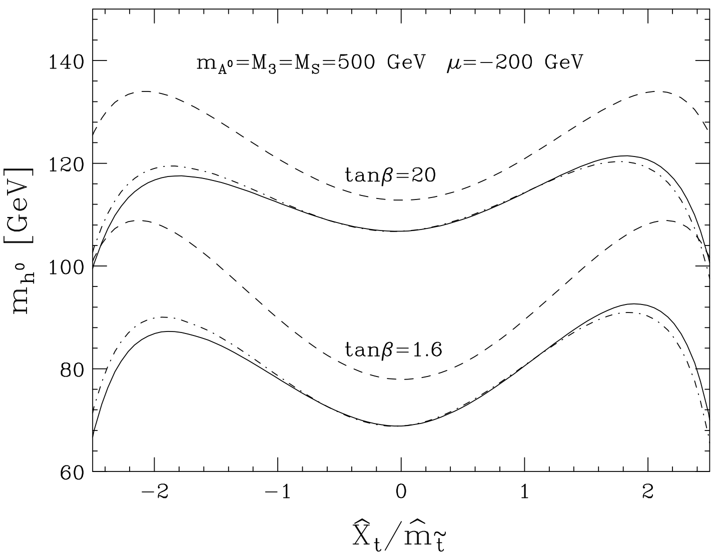

Figure 1: Higgs boson mass m_{h^0} versus $\widehat{X}_t/\widehat{m}_{\tilde{t}}$. The solid lines and dotdashes correspond to the two-loop Higgs boson masses calculated from the RG-improved mass formula eq. (4.4), with and without including the two-loop finite threshold corrections. One-loop masses are shown in dashes for reference.

the top-quark mass scale m_t in the SM, which is the effective theory below $m_{\tilde{t}}$. The coupling in front of this logarithmic term has to be evaluated at the intermediate scale $Q_t = \sqrt{m_t m_{\tilde{t}}}$, choice which automatically takes into account higher order effects. Having computed the two-loop results, we can also say something on the scale at which the masses entering the logarithm should be evaluated, although not in an unambiguous way. If we insist in evaluating them at the same scale, that scale turns out to be $Q_{\tilde{t}} = Q'_t = m_t$. If, on the other hand, we prefer to keep $Q_{\tilde{t}} = m_{\tilde{t}}$, then $Q'_t = (m_t m_{\tilde{t}}^2)^{1/3}$. The finite non-logarithmic 'one-loop' term is interpreted as a threshold correction for the quartic Higgs coupling at the scale $m_{\tilde{t}}$, at which the SM is matched to the MSSM. In accordance to this, all the parameters in this term are evaluated at the threshold scale $Q_{\rm th} = m_{\tilde{t}}$. Then, the remaining two-loop finite terms give a two-loop contribution to this threshold correction.[7] Eq. (4.4) can be slightly modified to automatically include the logarithmic terms of order $\mathcal{O}(\alpha_t^2)$ (and higher) if we refine eqs. (3.9), (4.1), (4.2) and (4.3) to take into account $\mathcal{O}(\alpha_t)$ corrections.

We show in figure 1 the numerical result from the RG-improved Higgs boson mass formula eq. (4.4). The two-loop finite threshold corrections are generally small for small and moderate mixing parameter $\widehat{X}_t/\widehat{m}_{\tilde{t}}$. For large mixing $\widehat{X}_t/\widehat{m}_{\tilde{t}} \gtrsim 2$, however, the two-loop threshold corrections can contribute corrections of the size of $\sim 3\,$GeV.

[7] Actually, an alternative way of computing this two-loop threshold correction is the following. Start with the expression for the effective potential in eq. (3.2) and make use of eq. (3.9) to trade between the running masses in MSSM and SM. Expand then the potential in powers of m_t. In that expansion, the finite term of order m_t^4 gives the correction to the Higgs quartic coupling, in accordance with our final result eq. (4.4).

In terms of the parameter $X_t(Q_{\rm th})/m_{\tilde{t}}(Q_{\rm th})$, we can obtain the refined condition for maximal mixing as

$$\left[\frac{X_t(Q_{\rm th})}{m_{\tilde{t}}(Q_{\rm th})}\right]_{\rm max} = \pm\sqrt{6} + \frac{5\alpha_s}{3\pi}\,, \tag{4.5}$$

which corresponds to a 2% shift to the position of maximal mixing. This condition can be rewritten in terms of OS quantities as

$$\left[\frac{\widehat{X}_t}{\widehat{m}_{\tilde{t}}}\right]_{\rm max} = \pm\sqrt{6}\left[1 - \frac{\alpha_s}{\pi}\left(3 + \ln\frac{m_{\tilde{t}}^2}{m_t^2}\right)\right] + \frac{\alpha_s}{\pi}\,. \tag{4.6}$$

We are now ready to compare with the formulae presented in [5] using the RG-improved one-loop potential (some two-loop input was used in the last paper in [5]). For zero squark-mixing, the leading and next-to-leading logarithmic terms in eq. (3.13) agree with [5] but those papers also gave the RG-improved one-loop threshold correction for non-zero top-squark mixing case, which is (omitting the $\mathcal{O}(\alpha_t^2)$ terms)

$$\frac{3\overline{m}_t^4(m_t)}{2\pi^2 v^2}\left[\frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12 m_{\tilde{t}}^4}\right]\left[1 - \frac{4\alpha_s}{\pi}\ln\frac{m_{\tilde{t}}^2}{m_t^2}\right]. \tag{4.7}$$

In this formula the X_t parameter is implicitly evaluated in the MSSM at the threshold scale, exactly as in our formula eq. (4.4), and agreement between the two is found after expressing $\overline{m}_t(m_t)$ in terms of $\overline{m}_t(Q_{\rm th})$. However, the two-loop finite terms included in eq. (4.4) can not be reproduced by the RG-improved one-loop effective potential approach.

To summarize, our analytical expression for $m_{h^0}^2$ agrees with those obtained by explicit two-loop calculation [10] and RG-improved one-loop effective potential approach [5] where they overlap. We note that parameters in different models (i.e. MSSM and SM) and renormalization schemes have been used in previous studies, our discussions should have resolved any possible confusion.

5. Conclusions and outlook

We have used the knowledge of the MSSM effective potential up to two-loop $\mathcal{O}(\alpha_s\alpha_t)$ to extract a simple analytical approximation to the mass m_{h^0} of the lightest CP-even Higgs boson in the simple case of a single supersymmetric threshold significantly higher than M_Z. We have considered the case of arbitrary $\tan\beta$ and non-zero mixing in the top-squark sector. We have derived a RG-improved formula to resum large logarithmic corrections, achieving a particularly simple and illuminating final result.

Our results agree with previous analyses based on the RG-improved one-loop effective potential [5] and diagrammatic calculation [8] to the order at which such

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studies were performed. By doing this we clarify the relation between different approaches and identify the two-loop origin of the discrepancies between [5] and [8] for large values of the top-squark mixing. This difference can be attributed to a two-loop threshold correction, most easily calculable in the effective potential approach, as we have shown. We emphasize that in applying two-loop mass correction formulae, one should be particularly careful about the parameters entering the one-loop formulae; we have done this by differentiating running and OS parameters in our mass correction formulae, eqs. (3.7), (3.13) and (4.4).

The combined use of the three different techniques (effective potential, diagrammatic calculation and RG-resummation) is very powerful and should be applied to compute the still missing $\mathcal{O}(\alpha_t^2)$ corrections to $m^2_{h^0}$, which can be similar in magnitude to those analyzed in this paper. Phenomenological analyses done with the expressions currently given in the literature are accurate only up to the inclusion of these $\mathcal{O}(\alpha_t^2)$ corrections, which one could expect to give a shift in m_{h^0} not greater than 5 GeV, but quite interestingly, we expect that this shift goes in a direction opposite to that of the two-loop $\mathcal{O}(\alpha_s\alpha_t)$ corrections. The leading and next-to-leading logarithmic corrections may be obtained by a RG resummation approach, but the finite non-logarithmic corrections can only be extracted by a direct two-loop calculation to the order $\mathcal{O}(\alpha_t^2)$, e.g., from effective potential or from explicit diagrammatic calculation.

Note added. A comparison between the diagrammatic and RG results for $m^2_{h^0}$ is mentioned in M. Carena, S. Heinemeyer, C.E.M. Wagner and G. Weiglein in `hep-ph/9912223`, which appeared at the time of submitting this paper.

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